Welcome

Welcome to the Postgraduate Group Theory Conference in Cambridge. This year, the conference is organised by

- **Ben Fairbairn**, University of Birmingham,
- **Alex Frolkin**, University of Cambridge,
- **Darren Kenneally**, University of Cambridge.

These are the people you should talk to if you have any queries.

This is the ninth such event, and in previous years it was held at the following universities:

- University of Southampton, 2006,
- University of Newcastle, 2005,
- University of Warwick, 2004,
- University of Oxford, 2003,
- University of Birmingham, 2002,
- Imperial College, 2001,
- Queen Mary and Westfield College, 2000,
- Royal Holloway, 1999.

If you are interested in hosting the conference next year, please speak to the organisers.

We hope that you will find the event interesting and enjoyable!

Conference arrangements

The conference will be held in the Centre for Mathematical Sciences (CMS). All talks will be held in meeting room 2 (MR2), and tea and coffee will be served upstairs, in the central core (cafeteria), during the breaks. An overhead projector, laptop and data projector, and chalk boards will be available.

Accommodation arrangements

Most participants will be staying in Churchill College, a short walk from the CMS. Accommodation will be available from 14:00 on Wednesday (unless you're arriving the day before) until 9:00 on Friday. We will take you to Churchill College after the last talk on Wednesday, and you will be able to leave your bags in the CMS before that. If you're planning to go to Churchill alone, you will need to collect your keys from the porters' lodge. When leaving on Friday morning, take all your things with you, and return your key to the porters' lodge.

Catering arrangements

Breakfast for those staying in Churchill College will be served there between 8:00 and 9:00 on Thursday and Friday mornings. Lunch on Thursday and Friday will be served in Wolfson Court, next to the CMS, between 12:30 and 13:30. Dinner on Wednesday will be served between 18:30 and 19:30, also in Wolfson Court. You will need to wear your badges in order to be served at Wolfson Court. The conference dinner will take place at 19:00 on Thursday evening, in New Hall.

Acknowledgements

The conference is funded by the London Mathematical Society. We are very grateful for their continuing support, without which the conference would not have been possible.

We would like to extend our thanks to the following people for their help, advice, support, and assistance: Sally Lowe, Chris Brookes, Geoffrey Grimmett, Michelle Scurll, Helen Innes, Mick Young, Ruth Allwood, Almarie Ehlers, and Maureen Hackett. We are grateful to the CMS for allowing us the use of their facilities free of charge.

We would also like to thank our guest speakers for agreeing to give the plenary talks.

Of course, thanks are also due to all the participants of the conference, without whom the conference would not have happened!

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Guest speakers

The guest speakers will be Professor Robert Wilson (Queen Mary, University of London) and Dr Peter Neumann (University of Oxford).

Robert Wilson — *The Ree groups: an elementary approach*

The classification of the finite simple groups is generally acknowledged to be one of the highlights of 20th century mathematics. Unlike Fermat's Last Theorem, however, it cannot be stated succinctly in terms the average mathematics undergraduate can understand, since it requires for its statement definitions of all the finite simple groups. Some of these are decidedly difficult to define in a useful way: the Ree groups are a case in point, and have languished in a certain amount of obscurity as a result.

I shall present a new approach to defining the Ree groups, which I hope is both elementary and illuminating. It dispenses entirely with the Lie-theoretic machinery, and uses instead the octonion algebra (sometimes known as the Cayley numbers) and the exceptional Jordan algebra (of three by three Hermitian matrices over the octonions) to lead directly to being able to write down generating matrices (by hand) and to prove that the resulting groups have all the required properties.

Peter Neumann — *Frobenius Groups, Past, Present and Future*

Frobenius groups can be described in many different ways. They were invented by a French mathematician called Eduard Maillet in his doctoral thesis of 1892. They were studied by Burnside, Frobenius, Zassenhaus, Higman, Thompson, and others, and much is now known about the finite Frobenius groups. Even so there is one major open problem. And when it comes to infinite groups there are many more. This lecture is intended as a survey, part historical, part mathematical, part inspirational.

Programme

	Wednesday	Thursday	Friday
9:00– 9:30		Darren Kenneally	Jia Lun Huang
9:30–10:00		David Stewart	David Craven
10:00–10:30		Daniel Gold	Jonathan Dixon
10:30–11:00		Anton Evseev	Robert Heffernan
11:00–11:30		*Coffee*	*Coffee*
11:30–12:00		Stuart Alder	Claude Marion
12:00–12:30	*Registration*	Ben Fairbairn	Jonathan Ward
12:30–13:00		*Lunch*	*Lunch*
13:00–13:30			
13:30–14:00		Marianne Johnson	Peter Neumann
14:00–14:30	Robert Wilson	Seamus Kelly	
14:30–15:00		Tom McKay	
15:00–15:30	Maura Clancy	Sarah Astill	
15:30–16:00	Elliot Costi	*Coffee*	
16:00–16:30	*Coffee*	Liam Naughton	
16:30–17:00	John MacQuarrie	David O'Keeffe	
17:00–17:30	Mohan Panchanathan		
17:30–18:00			
18:00–18:30			
18:30–19:00	*Dinner*		
19:00–		*Conference dinner*	

Abstracts

Stuart Alder — *q-complexes and modular homology representations*

A q-complex is a natural generalisation of an ordinary simplicial complex. These structures do appear widely, but of particular interest are the q-complexes that are contained in the subgroup lattices of certain groups. This presentation introduces the concept of these q-complexes and the role they can play in the explicit construction of homology representations in positive characteristic for finite groups. An application of these ideas to the Mathieu groups is discussed.

Sarah Astill — *Using amalgams to recognise finite simple groups*

We will see how finite simple groups can be recognised up to isomorphism from their so-called p-local structure for some prime p. Given a finite group G and partial information about some subgroup structure related to the normalisers of certain non-trivial p-subgroups we aim to understand some of the local structure inside G. We then consider a combinatorial object, the coset graph, and use it to recognise the possible isomorphism types of G. In particular we will see a characterisation of the finite simple group $G_2(3)$ from its 3-local structure.

Maura Clancy — *Introduction to the $\mathrm{K}(\pi;1)$-conjecture*

Let G be a finitely presented group. A classifying space $\mathrm{B}(G)$ for G is a connected CW-space whose fundamental group $\pi_1(\mathrm{B}(G))$ is isomorphic to G and whose higher homotopy groups are all trivial. That is, $\mathrm{B}(G)$ is an Eilenberg–MacLane space of type $\mathrm{K}(G;1)$. We can view $\mathrm{B}(G)$ as an orbit space X/G where X is any contractible space admitting a fixed-point free action of G. For example, the free abelian group $\mathbb{Z}^2$ of rank two acts freely on the Euclidean plane $\mathbb{R}^2$, and the orbit space $\mathbb{R}^2/\mathbb{Z}^2$ is the torus. Witold Hurewicz proved that the homotopy type of such spaces is uniquely determined by G; thus homotopy dependent algebraic invariants of a space, such as homology groups, become group invariants. It is reasonable therefore to think of them as homology groups of G. Thus we have:

$$\text{if } \mathrm{B}(G) \text{ is a } \mathrm{K}(G;1)\text{-complex then } \mathrm{H}_*(G) \approx \mathrm{H}_*(\mathrm{B}(G)).$$

The $\mathrm{K}(\pi;1)$-conjecture for an Artin group π claims that a certain finite-dimensional space is a $\mathrm{K}(\pi;1)$. It is known to be true for spherical Artin groups. This talk will use some concrete examples to introduce the concepts outlined above.

Elliot Costi — *Writing an element of $\mathrm{SL}(d,q)$ in its non-natural representation as a word in its generators*

Consider $E \leqslant \mathrm{SL}(n,q)$, where $n \geqslant d$ and suppose that E is isomorphic to $\mathrm{SL}(d,q)$. In this talk, I will discuss an algorithm that writes an arbitrary element of E as a word in a standard generating set.

David Craven — *Algebraic modules and the Heller operator*

There are two naturally-occurring binary operations on the set of all representations of a given group: direct sum; and tensor product. The first of these is completely understood, but the tensor product operation is still rather mysterious, in the sense that, given two representations, ρ and τ, we can't really understand the decomposition of $\rho \otimes \tau$.

The concept of an algebraic representation is completely analogous to that of an algebraic number: it satisfies a polynomial equation in $\oplus$ and $\otimes$. If the characteristic of the field over which representations are taken is 0, then every representation is algebraic. However, if we work with modular representations, this is not always the case.

Finally, the Heller operator is a concept borrowed from topology. It turns out that this operator is a bijection on the set of non-projective indecomposable representations. We can therefore ask questions about where this Heller operator sends algebraic representations. I will answer these.

Jonathan Dixon — *Verma modules*

Verma modules are the universal highest weight modules for Lie algebras. I will introduce them and briefly describe some results about them in characteristic p and their quantised analogues.

Anton Evseev — *Counting p-groups: the PORC conjecture*

G. Higman conjectured that the number of groups of order p^n is, for fixed n, a well-behaved ('polynomial on residue classes') function of the prime p. I will explain what this means and will outline a proof of this conjecture for a certain small family of groups.

Ben Fairbairn — *Squishing a Leech*

The Conway group, $\cdot 0$, is integral to almost any discussion of the sporadic simple groups — it provides the starting point for the construction of many larger ones (including the infamous Monster) and even twelve of the smaller ones arise as sections of $\cdot 0$. It is thus immediate why being able to describe elements of $\cdot 0$ uniformly is so important.

In this talk I shall describe a standard 'short form' for elements of the group and a programme I have written to multiply together elements in this group.

Daniel Gold — *G-complete reducibility*

We present an overview of a notion that generalises the representation theory of algebraic groups.

Robert Heffernan — *On the sum of character degrees of a finite group*

Let $\mathrm{T}(G)$ be the sum of the degrees of the irreducible complex representations of a finite group G and suppose that p^n divides $|G|$ where p is a prime number. I produce a lower bound for $\mathrm{T}(G)$ in general. For n at most 6 I briefly look at better bounds for $\mathrm{T}(G)$.

Jia Lun Huang — *Factorisation of tensor products*

An introduction to the computational problems of factorising tensor products of matrices, polynomials and multisets from an abelian group. These factorisation problems are related. Solving one of these problems may rely on the solution of another.

Marianne Johnson — *Standard tableaux and Klyachko's Theorem on Lie representations*

Let $n \in \mathbb{N}$ and let $\lambda = (\lambda_1, \ldots, \lambda_k)$ be a partition of n. That is, $\lambda_1 \geqslant \cdots \geqslant \lambda_k > 0$ and $\lambda_1 + \cdots + \lambda_k = n$. The Young diagram corresponding to λ is a collection of n boxes arranged in left justified rows with λ_i boxes in the i^{th} row. A standard tableau of shape λ is a numbering of the Young diagram of λ with the numbers from $\{1, \ldots, n\}$ in such a way that the entries strictly increase along every row and down every column. We say that an entry i is a descent in a standard tableau T if the entry $i + 1$ occurs in any row below the row in which i appears. The sum of all descents in T is called the major index of T. For example, $\lambda = (5, 3, 2, 2, 1)$ is a partition of 11. Shown below is a standard tableau T of shape λ. The descents of T are 2, 6, 9 and 10. Hence, the major index of T is $2 + 6 + 9 + 10 = 27$.

1	2	5	6	9
3	4	10		
7	8			
11				

We show that for all partitions λ of $n \geqslant 3$ ($\lambda \neq (2^2)$, (2^3), (1^n), (n)) there exists a standard tableau of shape λ with major index coprime to n [1]. By a result of Kraśkiewicz and Weyman [3], this provides a new purely combinatorial proof of Klyachko's Theorem [2] on Lie representations of the general linear group.

[1] M. Johnson. *Standard tableaux and Klyachko's Theorem on Lie representations.* Journal of Combinatorial Theory, Series A, **114** (2007), no. 1, 151–158.

[2] A. A. Klyachko. *Lie elements in the tensor algebra.* Sibirsk. Mat. Zh., **15** (1974) 1296–1304 (Russian). English translation: Siberian J. Math. **15** (1974), 914–921.

[3] W. Kraśkiewicz, J. Weyman. *Algebra of coinvariants and the action of a Coxeter element.* Bayreuth. Math. Schr., **63** (2001), 265–284. (Preprint, 1987).

Seamus Kelly — *Orbit polytopes*

A polytope is the convex hull of a subset of $\mathbb{R}^n$. We are interested in orbit polytopes which are polytopes which arise from finite groups G. We will describe the face lattice of the orbit polytope in the case where G is the symmetric group S_n. This information can be used to calculate the combinatorial structure of the orbit polytope in the case where G is the even subgroup of S_n.

Darren Kenneally — *Weight watching*

I will carry out a calculation using root strings and root nets.

John MacQuarrie — *Profinite groups are big (or small)*

The category of profinite groups sits within the category of topological groups. Its objects are appropriate limits of certain systems of finite groups, where the finite groups are given the discrete topology. Profinite groups share many of the properties of finite groups, but one property they do not share in general is that of being finite. In fact, using the awesome (compact) topological properties of profinite groups, we will show that if a profinite group isn't small (finite) then it isn't medium (countable), so by exhaustion must be big.

Claude Marion — *Finite simple groups of Lie type of small rank and hyperbolic triangle groups*

We start with the following result proved by Liebeck and Shalev.

Theorem 1 *Let Γ be a Fuchsian group of genus $\geqslant 2$ and let G be a finite simple group. Then the probability that a randomly chosen homomorphism in $\mathrm{Hom}(\Gamma, G)$ is an epimorphism tends to 1 as $|G| \to \infty$.*

They also conjectured that if Γ is a Fuchsian group of genus 0 or 1 and G is a finite simple classical group of sufficiently large rank, then Theorem 1 holds.

We ask the natural question: does the conjecture still hold if Γ is a Fuchsian group of genus 0 and G is a finite simple group of Lie type of small rank?

Let p_1, p_2, p_3 be three prime numbers with $1/p_1 + 1/p_2 + 1/p_3 < 1$, and set Γ to be the hyperbolic triangle group

$$T_{p_1,p_2,p_3} = \langle x,\, y : x^{p_1} = y^{p_2} = (xy)^{p_3} = 1\rangle.$$

This is a Fuchsian group of genus 0. Let $\mathcal{F}$ be an infinite family of finite simple groups of Lie type of small rank. We consider the following two problems.

Problem 1 *Let $G \in \mathcal{F}$. When is there an epimorphism from T_{p_1,p_2,p_3} to G? In other words, when is G a (p_1, p_2, p_3)-group?*

Problem 2 *Let $\mathcal{F}(p_1, p_2, p_3) = \{G \in \mathcal{F} : G$ is a (p_1, p_2, p_3)-group$\}$. If $\mathcal{F}(p_1, p_2, p_3) \neq \emptyset$, take $G \in \mathcal{F}(p_1, p_2, p_3)$. What is the probability that a random homomorphism in $\mathrm{Hom}(T_{p_1,p_2,p_3}, G)$ is an epimorphism? If $\mathcal{F}(p_1, p_2, p_3)$ is infinite, what is the limit of this probability as $|G| \to \infty$?*

We try to attack those problems using a simple approach that involves character theory. We give an illustration by studying the following families: $\mathcal{F}_1 = \{L_2(q)\}$, $\mathcal{F}_2 = \{U_3(q)\}$, $\mathcal{F}_3 = \{L_3(q)\}$.

Tom McKay — *Plethysm conjectures of Stanley and Foulkes*

Let $N = N_\lambda$ denote the normaliser of the Young subgroup S_λ in a symmetric group G. We introduce the module $\mathbb{1}_N^G$ and go on to study the relationship between $\mathbb{1}_N^G$ and $\mathbb{1}_{N'}^G$, where N' is the normaliser of $S_{\lambda'}$.

Liam Naughton — *The subgroups of S_{13}*

The study of the symmetric group forms one of the oldest areas of group theory. In this short presentation I will present an algorithm which produces the conjugacy classes of subgroups of a symmetric group from the corresponding conjugacy classes of subgroups of the alternating group. This algorithm was recently used to produce the conjugacy classes of subgroups of S_{13}. I will also highlight some interesting properties of the conjugacy classes of subgroups of a symmetric group.

David O'Keeffe — *How groups are used in (co)homology theory*

There are many (co)homology theories in use but they all have something in common, that is the use of groups to shed some light on the particular structure one is working with. So, if one is working with topological spaces for example, (co)homology groups can be used to help classify topological spaces.

Mohan Panchanathan — *The Generalised Nottingham Group*

A pro-p group is the inverse limit of some system of finite p-groups. Impetus on current research comes from four main directions, namely number theory, classification of finite p-groups, the theory of infinite p-groups (pro-p completions) and the broader area of profinite group theory. Structural and classification theorems are mostly based on the study of known examples of pro-p groups. I will give a brief introduction to profinite and pro-p groups, survey the universe of countable pro-p groups and talk in some detail about the Generalised Nottingham Group, which is the group of formal power series in n variables, culminating in an explicit description of the lower central series and power structure of the group.

David Stewart — *G-complete reducibility*

Let G be an algebraic group defined over a field k. A subgroup H of G is said to be G-completely reducible (G-cr) if, whenever H lies in a parabolic subgroup P of G, it also lies in a Levi subgroup of that parabolic. This concept has been shown to be equivalent to Richardson's notion of strong reductivity and also has connections with so-called spherical buildings formulated by Tits. I hope to give an idea of what G-complete reducibility feels like and what I am working on.

Jonathan Ward — *Generating the finite simple groups with involutions (or not)*

I am working on finding out which of the non-abelian simple groups can be generated by 5 conjugate involutions whose product is the identity. I will give a brief overview of some of the methods one can use in problems such as this.